



Figure 6. What does this collection illustrate?

What might Figure 6 illustrate to students? It often is portrayed as illustrating $3/5$ as if it cannot illustrate anything else. A person could also see Figure 6 as illustrating that $1 \div \frac{1}{5} = 5$ —that within one whole there is one three-fifths and two-thirds of another three-fifths, or that $5 \div 3 = 1\frac{2}{3}$ —that within 5 is one 3 and two-thirds of another 3. Finally, they could see Figure 6 as illustrating $\frac{3}{5} \times \frac{5}{3} = 1$ —that five-thirds of (three-fifths of 1) is 1 (Figure 7).

If we see ●●●○○ as one collection, then ● is one-fifth of one, so ●●● is three-fifths of one.

If we see ●●● as one collection, then ● is one-third of one, so ●●●○○ is five-thirds of one.

If we see ● as one circle, then ●●●○○ is five circles, so ● is one-fifth of five, and ●●● is three-fifths of five.

If we see ● as one circle, then ●●● is three circles, so ● is one-third of three, and ●●●○○ is five-thirds of three.

Figure 7. Various ways to think about the circles and collections in Figure 6.

We rarely find texts or teachers discussing the difference between thinking of $3/5$ as “three out of five” and thinking of it as “three one-fifths.” How a student understands Figure 6 in relation to the fraction $3/5$ can have important consequences. When students think of fractions as “so many out of so many” they are puzzled by fractions like $6/5$. How do you take six things out of five?¹²

¹² We often hear teachers and teacher education students say “change $6/5$ to $1\frac{1}{5}$ and they’ll understand.” This misses the point. It is problematic if a student *must* change $6/5$ to $1\frac{1}{5}$, for it means that students cannot understand any situation in which they must see fractions as entailing a proportional relationship.