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Connecting Decimals and Other Mathematical Content

EXPERIENCES with many children in the middle grades indicate that they have poor decimal concepts and lack fundamental skills in working with decimal values. For example, when asked to identify which of 0.36 or 0.339 is greater, children frequently choose 0.339 because 339 is greater than 36. Other children reason that 0.339 is smaller because it has more decimal places, and they "know" that the values decrease as more decimal places are added. Likewise, children have poor number sense regarding decimal numbers. For example, when asked to round 0.487 to the nearest hundredth, they blithely use the standard rule "round up when the digit to the right is greater than 5," round 0.487 to 0.49, but never think of either 0.487 or 0.49 as being near 1/2 on the number line.

Children often develop misconceptions about decimals because instruction does not promote connections between decimals and other mathematical content. Decimal concepts and symbols need to be related to a variety of fraction ideas and to place value. Connections also need to link decimals with coin and dollar values, metric measurement, and percents.

This article describes instructional activities with decimals that enable children to make connections that are necessary for them to understand and use decimals meaningfully. Decimal concepts and symbols are related to fraction concepts and symbols and to place-value concepts and symbols. Extensive use is made of physical models, diagrams, and number lines. The goal is to enable students to develop decimals as part of an integrated network of number ideas by understanding (1) that decimals are a type of fraction with a different symbolism and (2) that decimals can be meaningfully compared, ordered, and related to common fractions by using fraction ideas and place-value ideas.

Decimals Are Base-Ten Fractions

Decimals can be thought of as parts of a whole, a whole that has been divided into 10, 100, 1000, or some other number of parts that is a power of 10. From this perspective, decimals are special fractions and can appropriately be called decimal fractions, since the word decimal means "based on ten." The following activity focuses on the part-whole aspect of decimal fractions. The special symbols for decimals are addressed in the subsequent activity.

A motivating and intriguing way to help students learn about decimal fractions involves using individually wrapped slices of cheese. Using one slice as the model for one whole, ask each student to cut his or her whole into ten equal-sized strips.

The plastic wrapper can be used as a cutting board (see fig. 1). Ask what number name should be given to each of the newly cut strips. Ask students to justify their answers in terms of the pieces of cheese in front of them. Focus on the idea that tenths have been made, since the whole has been cut into ten equal-sized parts. This activity uses children's ideas about fractions, and teachers can have students write fraction symbols (1/10) to match their verbal answers (one tenth). Students can then answer questions related to the number of tenths in five whole slices of cheese (fifty tenths) or in two and one-half slices of cheese (twenty-five tenths).

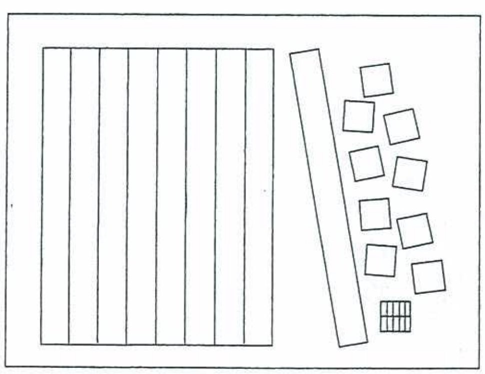

Fig. 1. Cutting a cheese slice to show decimals

After students can answer questions about the tenths strips and justify their answers, ask them to take one of the strips and cut it into ten equal-sized pieces, which will be little squares. Follow with the same type of questions as with the tenths: "What name should be given to the new pieces?" (Hundredths) "How do you know?" (One hundred of these pieces are needed to make the whole slice.) "What name would you give to three strips and two little squares?" (3 tenths and 2 hundredths, or 32 hundredths, or 32/100, since each of the three strips has 10 hundredths.)

Students seem to like the challenge of taking one of the hundredths squares and cutting it into ten equal pieces, thereby

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creating thousandths. The cutting gets tedious at this point, but it is not impossible. The relative sizes of tenths, hundredths, and thousandths become meaningful for students through this activity. They can see the three sizes! They can respond to questions about the values of combinations of the pieces. "What is the value of two strips, one little square, and three tiny pieces?" (213/1000, since each strip contains 100 thousandths, each little square contains 10 thousandths, and 3 extra thousandths are present.) When the discussion is completed, the children enjoy eating their decimal models. For example, ask the students to eat twenty-three hundredths of their original slice, then 3 tenths, 6 thousandths, and so on, until all the cheese has been eaten.

Decimal Connections with Place Value

In the following activity, children are challenged to extend the place-value notation they have been using for whole numbers to include symbolic representations for decimal fractions. Children manipulate decimal models on a place-value mat (fig. 2), verbalize the amounts shown, and later enter the appropriate numerals on a calculator. The materials needed for each pair of children are tagboard decimeter squares to represent the whole, strips 1 cm by 1 dm to represent tenths, squares 1 cm by 1 cm to represent hundredths, three-column place-value mats, and a calculator.

Ask the students to compare the strips with the large square and to name them in terms of the large square. Be sure that the students use two characteristics to justify that the strips are tenths: (1) they are equal in size and (2) ten of them make the whole square. Have one student in each pair place one large square in the left column of the place-value mat and one strip in the column immediately to the right. Ask everyone to verbalize the total represented (one and one-tenth) and discuss how to use numbers to write that amount (1 1/10).

Challenge the students with the following idea: "Is it possible to use '11' to stand for the value of the square and the strip? Let's use the '1' on the left to stand for the large square and the '1'on the right to stand for the strip." Students will object, of course, and state that '11' stands for eleven rather than one and one-tenth. One student will usually suggest putting a dot between the 1's (1.1), since many of them have seen decimals written elsewhere. Take this opportunity to challenge them to defend the use of this notation. Ask them to justify that this notation is consistent with the place-value notation that they learned earlier and to identify the meanings of the 1's and the dot. Their explanations should focus on the 1-to-10, right-to-left relationship between adjacent positions in the place-value system. They should also identify that the purpose of the dot, or decimal point, is to separate the whole from the parts.

To test their reasoning and understanding, suggest that if a whole square is cut into eight equal parts and one square and one-eighth of the square are placed on the mat, then 1.1 can be used to represent this amount also. Some children may need hints to help them determine that the reason 1.1 can be used for one and one-tenth but not for one and one-eighth is that a 1-to-10, right-to-left relationship must exist between adjacent positions in the standard place-value system. One hint would be to ask children how the values of the two 1's compare in each of the following numbers: 211, 112, and 1123. (In each example the 1 on the right is worth 1/10 of the 1 on its left.) Another hint would be to have children examine previously used place-value materials, such as base-ten cubes, rods, flats, and big cubes, that model this 1-to-10 relationship.

Following this investigation of the place-value system, the students will have a beginning understanding that 1.1 can be used for one and one-tenth because the tenths are one-tenth in value of the units to the immediate left. This idea will be reinforced throughout the remainder of this activity.

Have the student in each pair who is using the calculator enter 1.1 to match the value of the pieces on the place-value mat. Add one more strip to each mat, and have the student with the calculator enter the corresponding symbols to represent this action: [+] [.] [1] [=]. Have students verbalize the new total on their mats (one and two-tenths) and discuss the calculator representation of this expression (1.2). (The "1" stands for one whole and the "2" stands for the two tenths. The decimal point separates the whole from the parts, the tenths.) Have the students continue adding strips, or tenths, one at a time to their mats as their partners press the corresponding keys on their calculators, until a total of 1.9 is reached. Consider using the constant feature of many calculators to add 0.1 repeatedly by pressing [=] after each addition of a tagboard strip. However, many children will benefit from representing the entire action initially by pressing all four keys, [+] [.] [1] [=], each time.

At this point, add another strip and ask the students to name the total on their mats. (One and ten tenths.) Discuss what should be done with the ten strips. (Trade them in for a whole square.) Have students predict what the calculator will do when [+] [.] [1] [=] is entered. (It automatically "makes the trade" and the display reads simply 2.) Be sure to connect this trading to the cheese activity: it corresponds to reconnecting the ten cheese strips and exchanging them for a new slice. Also, ask the children to check the odometer of their parents' car to see what happens immediately after it changes from a 9 in the tenths place. (The ones digit advances by 1 and the tenths digit changes to 0.)

Continue adding tenths until the children can verbalize the amount on the place-value mats correctly and can predict

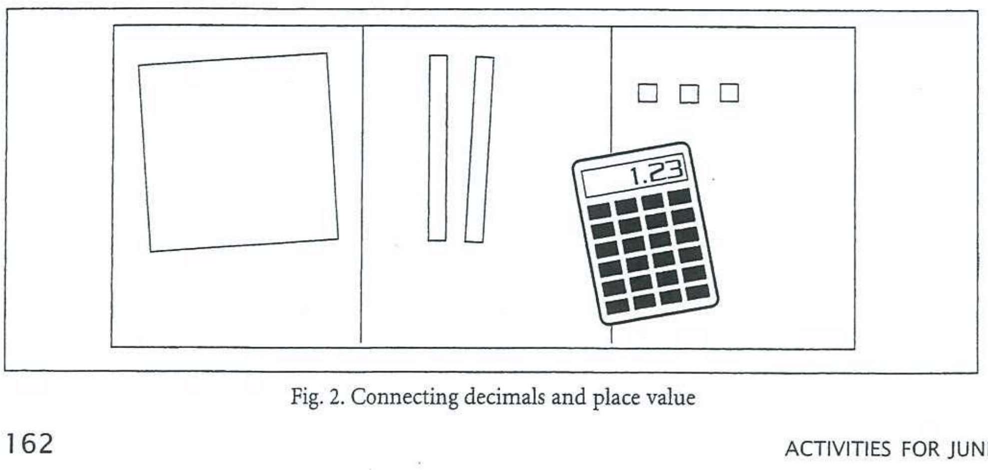

Fig. 2. Connecting decimals and place value